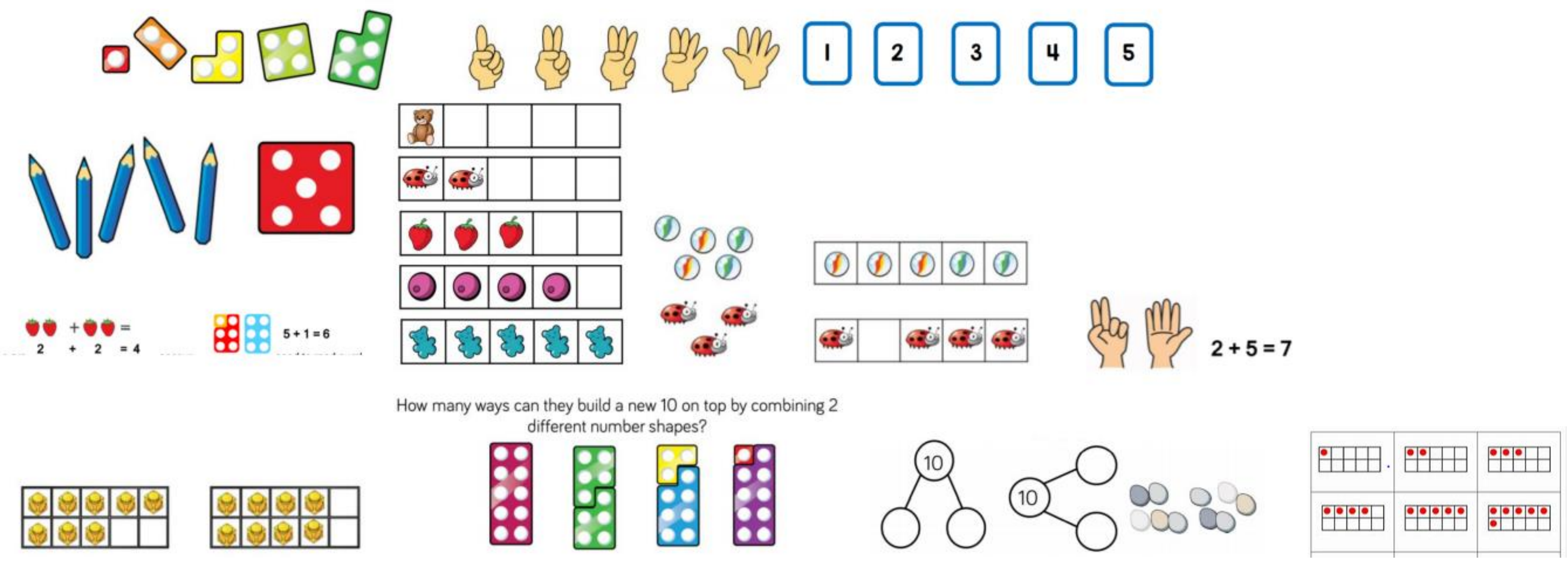
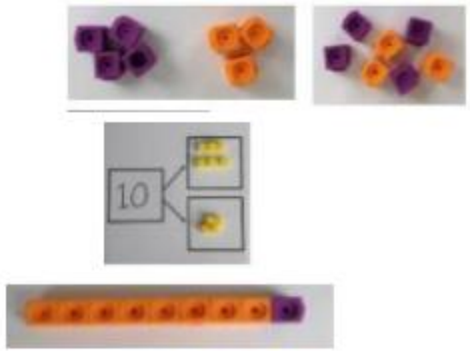
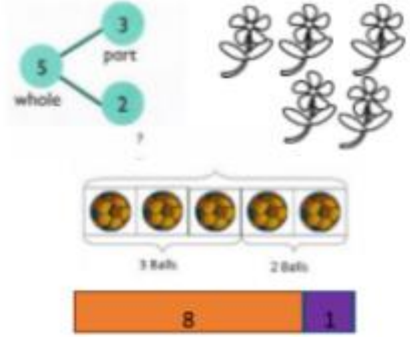
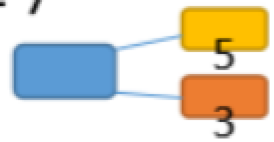
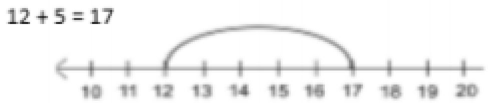
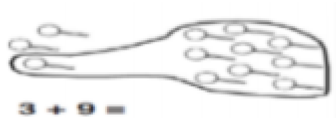
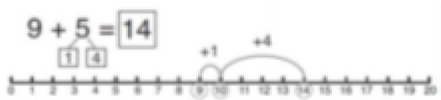
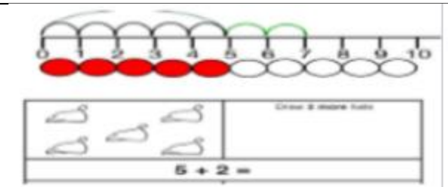
Nursery and EYFS Addition

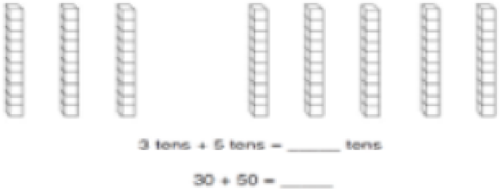
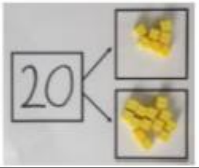
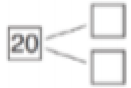
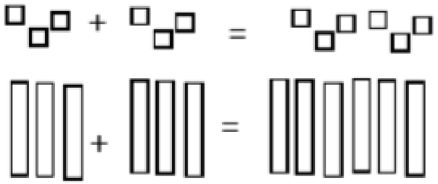
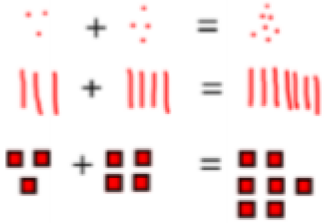
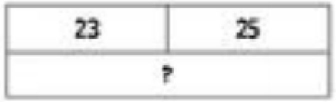
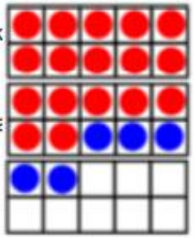
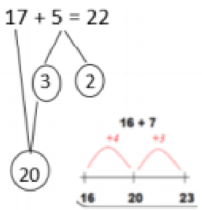
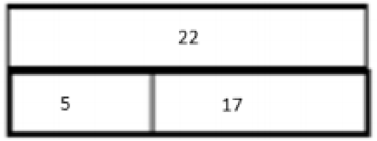
Objective and Strategy	Models and Guidance
Count reliably from 1-5, 1-10 and 1-20 Identify one more and one less Combine objects to add Find number bonds Add without counting	**Nursery**: Before addition can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting, number order and number recognition through practical activities and games. This is taught through child-initiated games such as Hide and Seek and I Spy. Children also learn how to count 1-1 (pointing to each object as they count) and that anything can be counted, for example, claps, steps and jumps. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. **EYFS**: Before addition can be introduced, children in EYFS build on concepts taught in Nursery. Children need to have secure knowledge of number in order to begin addition. Children are then introduced to the concept of addition through practical games, singing and activities. Fives and tens frames are used significantly to gain full understanding of addition. Children act out addition calculations to physically add two groups of objects together. This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definitions. Adults support children in recording their addition calculations in the written form on whiteboards.

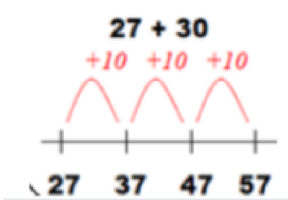
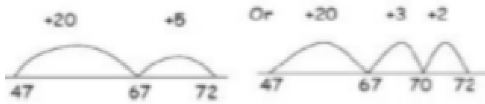
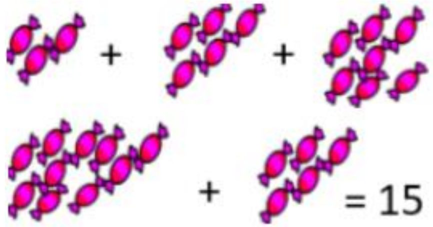


Year 1 Addition

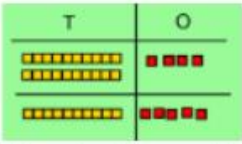
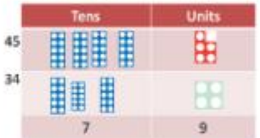
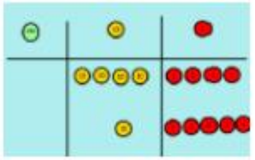
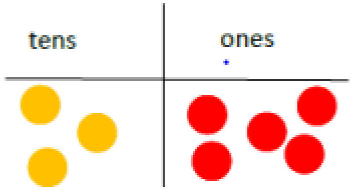
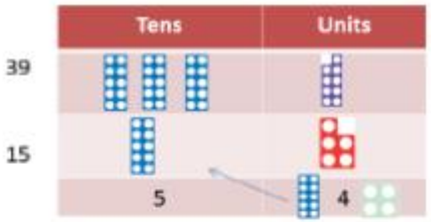
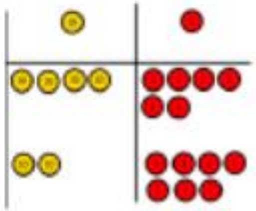
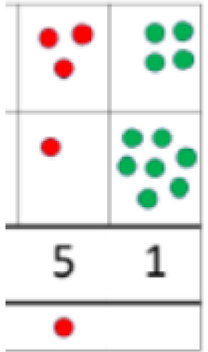
Objective and Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model.	Use part-part whole model. Use cubes to add two numbers together as a group or in a bar.  	Use pictures to add two numbers as a group or in a bar.  	Use the part-part whole diagram as shown below to move into the abstract. 4 + 3 = 7  10 = 6 + 4
Starting at the bigger number and counting on.	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer.  	Place the larger number in your head and count on the smaller number to find your answer. 12 + 5 = 17
Regrouping to make 10. **This is an essential skill for column addition later.**	Start with the bigger number and use the smaller number to make 10. Use ten frames.  6 + 5 = 11 	Use pictures or a number line. regroup or partition the smaller number using the part-part whole model to make 10.   	If I am at seven, how many more do I need to make 10? How many more do I add on now? 7 + 3 = 10
Represent and use number bonds and related subtraction facts within 20.	2 more than 5. 	 	Emphasis should be on the language. '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

<u>**Year 2 Addition**</u>

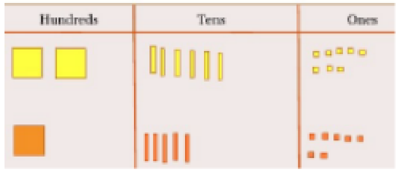
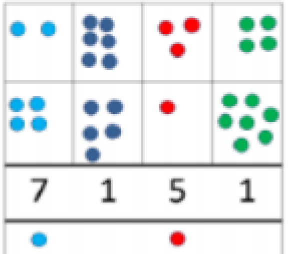
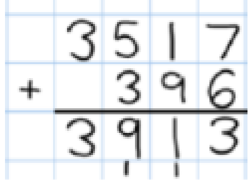
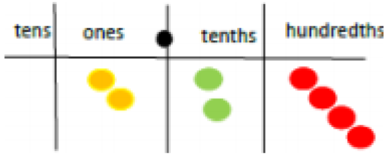
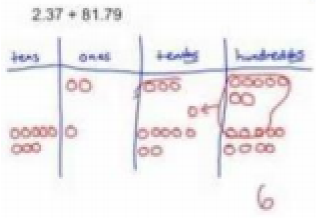
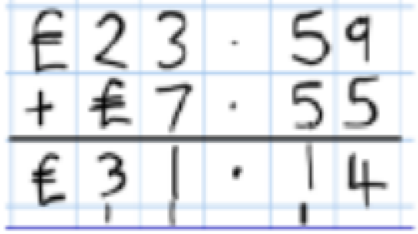
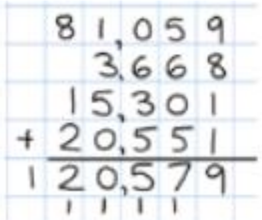
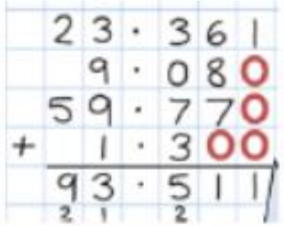
Objective and strategy	Concrete	Pictorial	Abstract
Adding multiples of 10	Model using dienes and bead strings 50 = 30 + 20 	Use representations for base ten. 	20 + 30 = 50 70 = 50 + 20 40 + ▭ = 60
Use known facts Part-part whole	Children explore ways of making numbers within 20. 	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		Children draw representation of H, T, O 	3 + 4 = 7 *Leads to* 30 + 40 = 70 *Leads to* 300 + 400 = 700
Bar model	3 + 4 = 7 	7 + 3 = 10 	 23 + 25 = 48
Add a two-digit number and ones	17 + 5 = 22 Use ten frames to mak Children explore the pa 17 + 5 = 22 27 + 5 = 32 	Use part-part whole and number line to model. 	17+ 5 = 22 Explore related facts 

Add a 2-digit number and a tens	Explore that the ones digit does not change. 25 + 10 = 35 		27 + 10 = 37 27 + 20 = 47 27 + $\boxed{}$ = 57
Add two 2-digit numbers	Model using dienes, place value counters and numicon.  25 + 47 = 72	Use number line and bridge ten using part whole if necessary. 	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers.	Combine to make 10 first if possible, or bridge 10 then add third digit.  7 + 3 = 10 10 + 2 = 12	Regroup and draw representation. 	Combine the two numbers that make / bridge ten then add on the third. 4 + 7 + 6 = $\boxed{10}$ + $\boxed{7}$ 10 = $\boxed{17}$

Year 3 Addition

Objective and strategy	Concrete	Pictorial	Abstract
Column addition – no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	Model using dienes or numicon.  Add together the ones first, then the tens.  Move to using place value counters.  44 + 15	Children move to drawing the counters using tens and one frame.  	Add the ones first, then the tens, then the hundreds. $\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$
Column addition with regrouping	Exchange ten ones for a ten. Model using numicon and place value counters.  	Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line.  	Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{ccccc} 20 & + & 5 & & \\ 40 & + & 8 & & \\ \hline 60 & + & 13 & = & 73 \end{array}$ $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\ 1 \end{array}$

Year 4 – 6 Addition

Objective and strategy	Concrete	Pictorial	Abstract
Year 4 – add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using place value grid. 	Continue from previous work to carry hundreds as well as tens.  Relate to money and measures.
Year 5 – add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4. Introduce decimal place value counters and model exchange for addition. 		
Year 6 – add several numbers of increasing complexity including adding money, measure and decimals with different numbers of decimal points.	As year 5	As year 5	Insert zeros for place value holders. 

<u>Nursery and EFYS Subtraction</u>

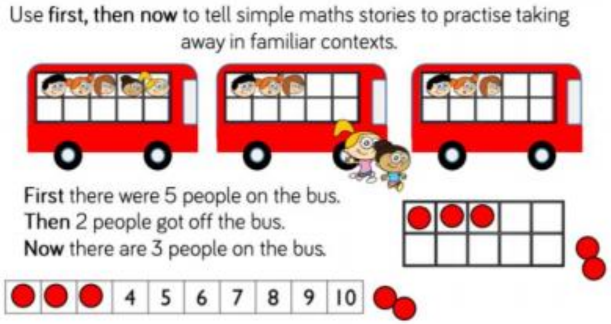
<table>
<tr><th>Objective and strategy</th><th>Models and guidance</th></tr>
<tr><td>Count back reliably from 1-5, 1-10 and 1-20
Identify one more and one less
Take away objects to subtract
Relate subtraction to number bonds
Subtract without counting</td><td>

<u>Nursery</u>: Before subtraction can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting backwards. This is taught through child-initiated games indoors and outdoors such as acting out counting songs and running races (children shouting "5,4,3,2,1,0 - GO!").

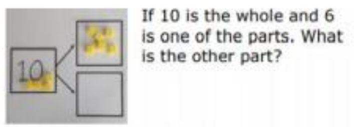
<u>EYFS</u>: Children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games, singing and activities. Children use 5 and 10 frames to ensure understanding is concrete. Children act out subtractions to physically subtract a number of objects from a group. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of 'less' by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definitions. Adults support children in recording their subtractions in the written form on whiteboards.

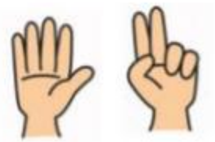
14-5=9

Make 14 on the ten frame or with different coloured cubes to represent the ten and the ones. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.



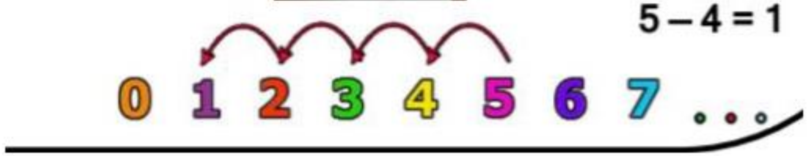
5 – 1 = 4





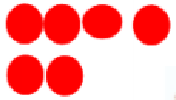
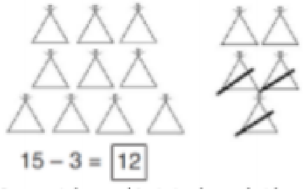
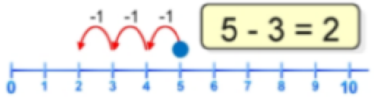
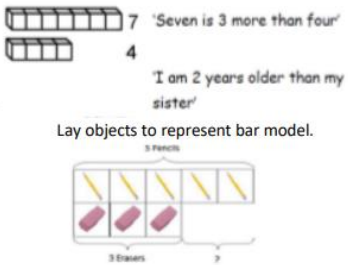
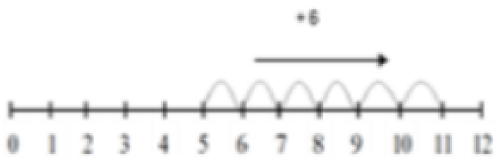
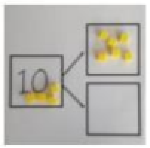
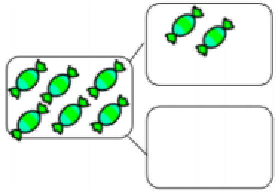
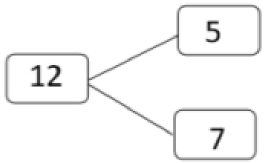
5 – 3 = 2

5 – 4 = 1



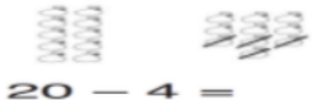
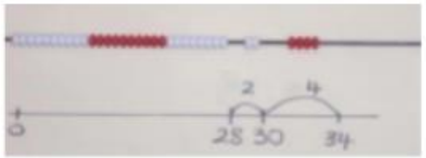
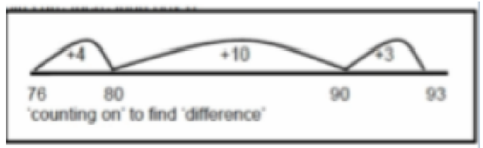
</td></tr>
</table>

Year 1 Subtraction

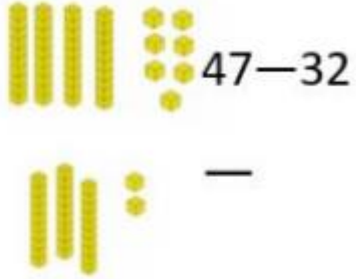
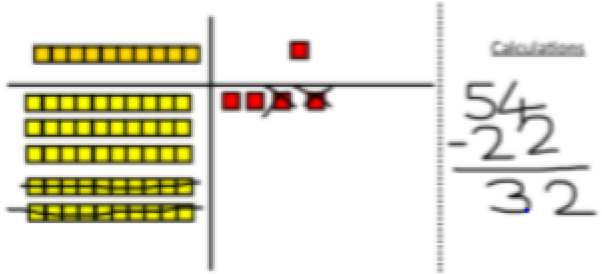
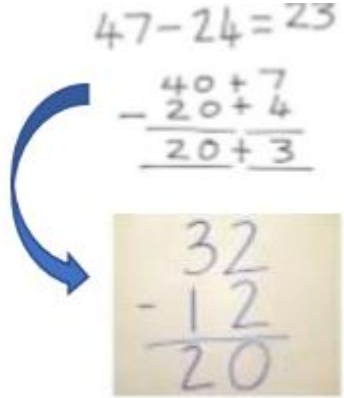
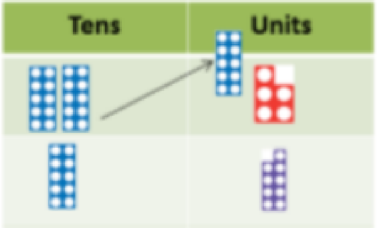
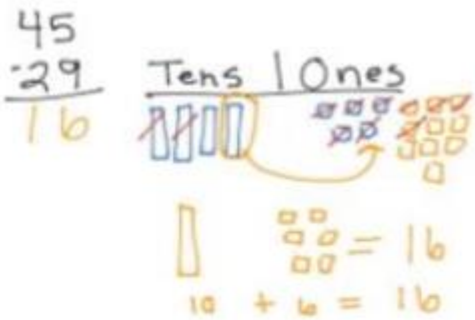
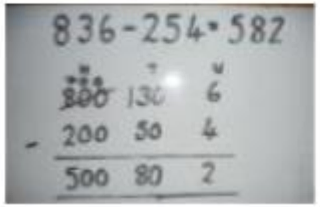
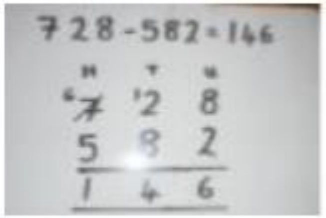
Objective and strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.   6 – 4 = 2 4 – 2 = 2	Cross out drawn objects to show what has been taken away. 	7 – 4 = 3 16 – 9 = 7
Counting back	Move objects away from the group, counting backwards. 	Count back in ones using a number line. 	*Put 13 in your head, count back 4. What number are you at?*
Find the Difference	Compare objects and amounts. 	Count on using a number line to find the difference. 	*Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?*
Represent and use number bonds and related subtraction facts within 20. Part-Part Whole model	Link to addition. Use part-part whole model to model the inverse.  10 – 6 = 4 If 10 is the whole and 6 is one of the parts, what's the other part?	Use pictorial representations to show the part. 	Move to using numbers within the part whole model. 
Make 10	Make 14 on the tens frame. Take 4 away to make ten, then take one more away so that you have taken 5.	Jump back 3 first, then another 4. Use ten as the stopping point.	*How many do we take off first to get to 10? How many are left to take off?* 16 – 8

	14 – 9 = 5 	13 – 7 = 6 	
Bar model	5 – 2 = 3 		8 | 2 10= 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

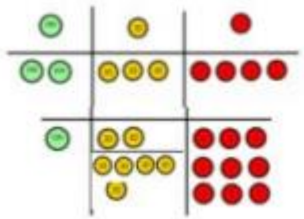
Year 2 Subtraction

Objective and strategy	Concrete	Pictorial	Abstract
Re-group a ten into ten ones.	Use a Place Value chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping.	Use Dienes to show how to partition the number when subtracting without regrouping.  34—13 = 21	Children draw representations of dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy. Progression should be crossing one ten, crossing more than one ten, crossing the hundreds	Use a bead bar or bead strings to model counting to the next ten and the rest. 	Use a number line to count on to next ten and then the rest. 	93 – 76 = 17

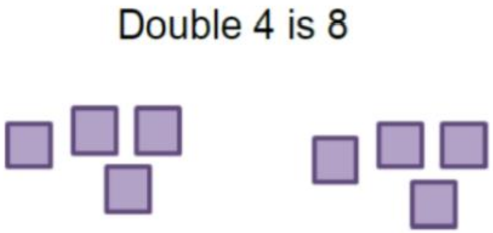
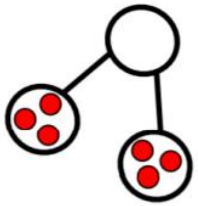
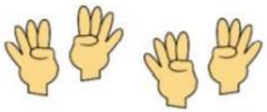
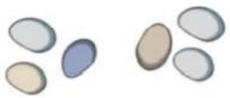
Year 3 Subtraction

Objective and strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping.	Use base 10 or Numicon to model. 	Draw representations to support understanding. 	Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping.	Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. 	Children may draw base ten or PV counters and cross off. 	Begin by partitioning into place value columns.  Then move to formal method. 

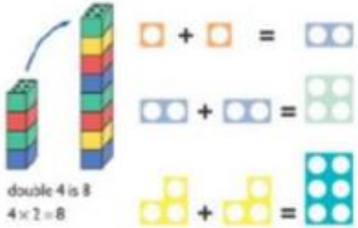
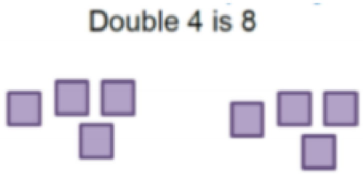
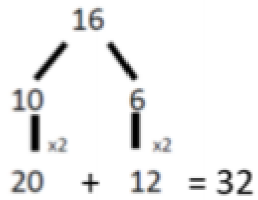
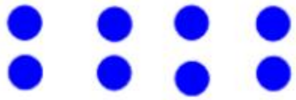
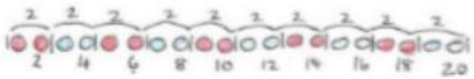
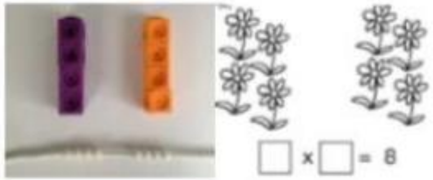
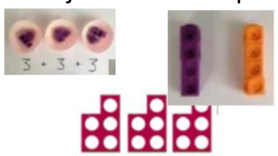
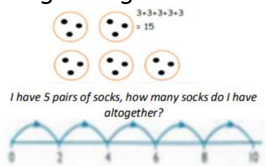
Year 4 - 6 Subtraction

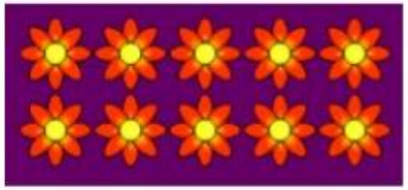
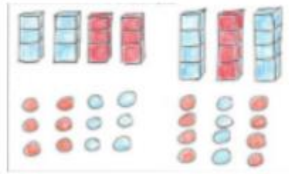
Objective and strategy	Concrete	Pictorial	Abstract
Year 4- Subtract with up to 4 digits. Subtracting tens and ones. Introduce decimal subtraction through context of money.	Model the process of exchange using Numicon, base ten and then move to place value counters. 234 - 179 	Children to draw place value counters and show their exchange (see year 3)	Show the exchange. $2754 - 1562 = 1192$
Year 5- Subtract with at least 4 digits, including money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal.	As year 4.	Children to draw place value counters and show their exchange—see Y3	Use zeros for placeholders. $31056 - 2128 = 28{,}928$ $7169.0 - 372.5 = 6796.5$
Year 6- Subtract with increasingly large and more complex numbers and decimal values.			$150{,}699 - 89{,}949 = 60{,}750$ $105.419\text{ kg} - 36.080\text{ kg} = 69.339\text{ kg}$

Nursery and EYFS Multiplication

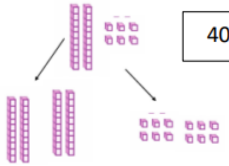
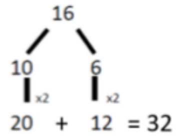
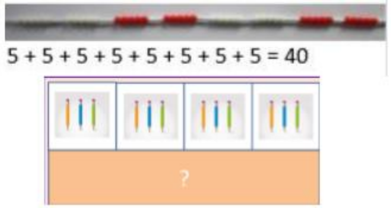
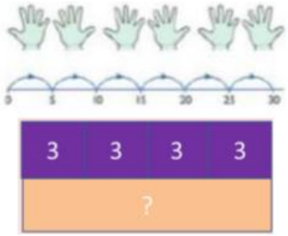
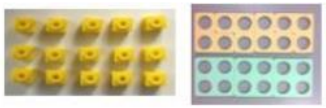
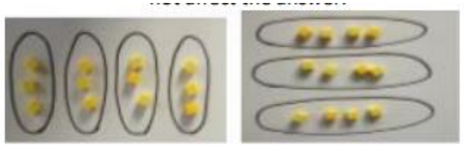
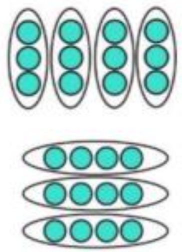
Objective and Strategy	Models and Guidance
To solve problems, including doubling.	**Nursery and EYFS**: By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer.           

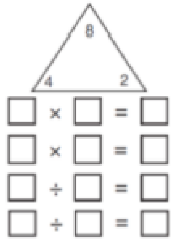
Year 1 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples.	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total.	Use manipulatives to create equal groups. 	Draw and make representations. 	2 x 4 = 8
Repeated addition.	Use different objects to add equal groups. 	Use pictorial including number lines to solve problems. *There are 3 sweets in 1 bag. How many sweets are there in 5 bags altogether?* 	Write repeated addition sentences to describe objects and pictures. 

Understanding arrays.	Use objects laid out in arrays to find the answers to 2 lots, 5 lots, 3 lots of 2 etc. 	Draw representations of arrays to show understanding. 	3 x 2 = 6 2 x 5 = 10

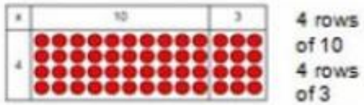
Year 2 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and place value counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30
Multiplication is commutative	Create arrays using counters, cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity. 	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

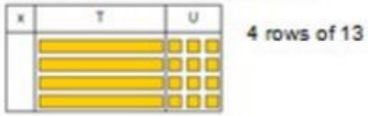
Using the Inverse. *This should be taught alongside division, so pupils learn how they work alongside each other.*	Use counters or cubes to introduce the inverse. 	Use number pyramids and pictorial representations to show related facts. 	Show all 8 related fact family sentences. 2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4

<u>**Year 3 Multiplication**</u>

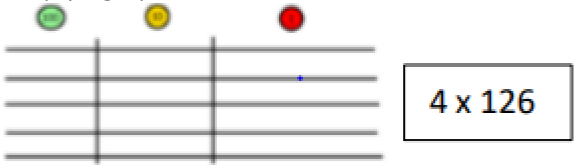
<table>
<tr>
<th>Objective and strategy</th>
<th>Concrete</th>
<th>Pictorial</th>
<th>Abstract</th>
</tr>
<tr>
<td>Grid method.</td>
<td>
Show the links with arrays to first introduce the grid method.



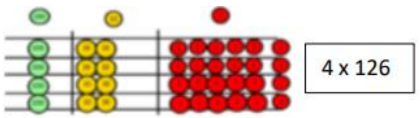
Move onto base ten to move towards a more compact method.



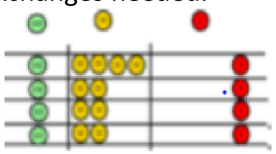
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



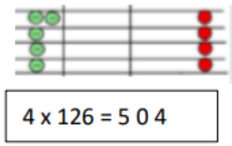
Fill each row with 126.



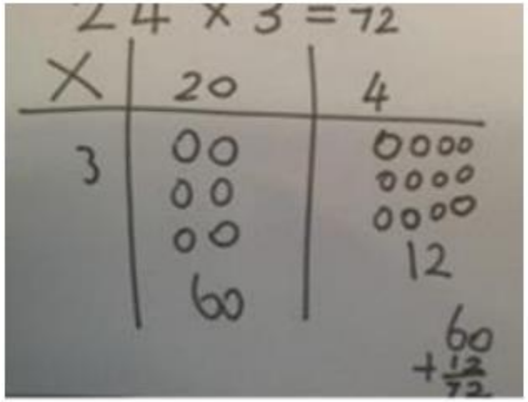
Add up each column, starting with the ones making any exchanges needed.



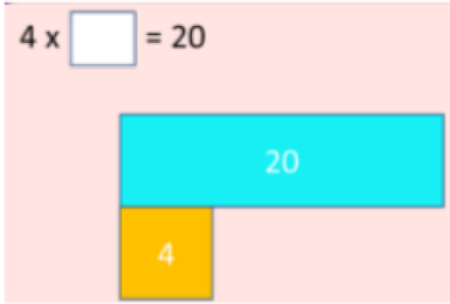
Give a final answer.



</td>
<td>
Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model is used to explore missing numbers.



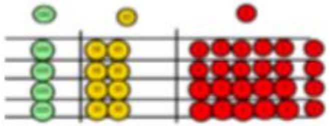
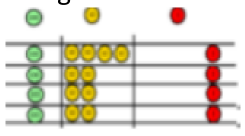
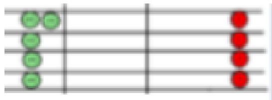
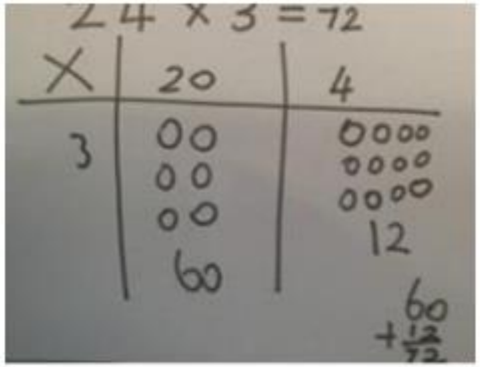
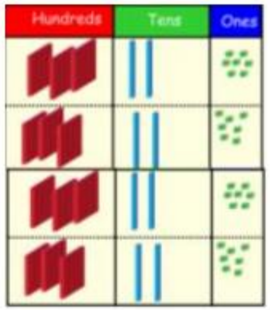
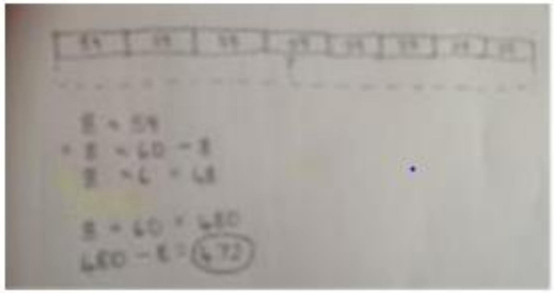
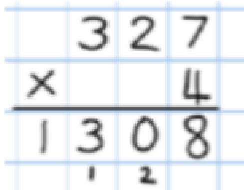
</td>
<td>
Start by multiplying with 1-digit numbers and showing clear addition alongside the grid.

<table>
<tr><td>x</td><td>30</td><td>5</td></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>

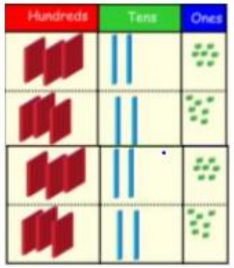
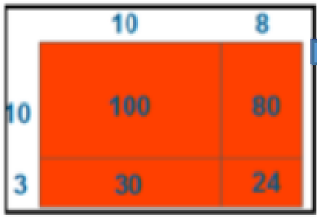
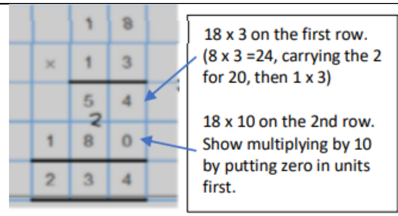
Moving forward, multiply by a 2- digit number showing the different rows within the grid method.

<table>
<tr><td></td><td>10</td><td>8</td></tr>
<tr><td>10</td><td>100</td><td>80</td></tr>
<tr><td>3</td><td>30</td><td>24</td></tr>
</table>
</td>
</tr>
</table>

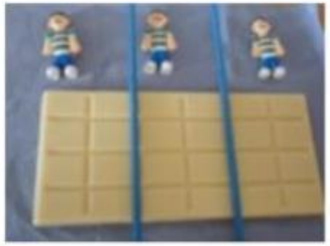
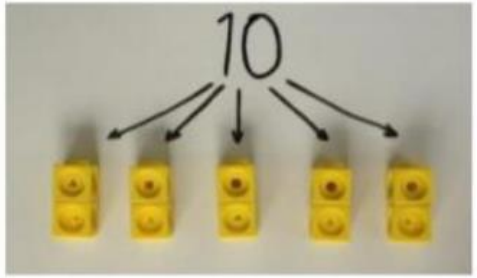
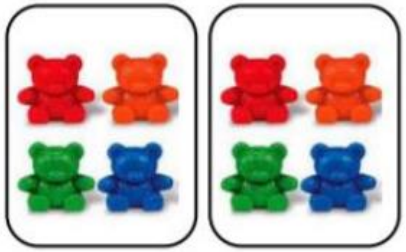
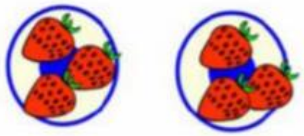
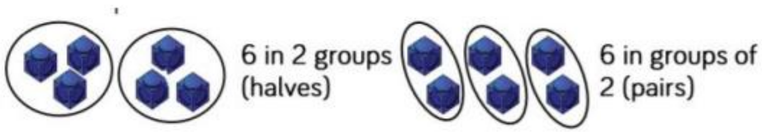
Year 4 Multiplication

<table>
<tr><th>Objective and strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3-digit numbers by 1 digit. (year 4 expectation)</td>
<td>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows. Fill each row with 126.

4 x 126
Add up each column, starting with the ones making any exchanges needed.

Give a final answer.

4 x 126 = 5 0 4</td>
<td>Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.

</td>
<td>Start by multiplying with 1-digit numbers and showing clear addition alongside the grid.
<table><tr><th>x</th><th>30</th><th>5</th></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642. It is important at this stage that they always multiply the ones first.

</td>
<td>The grid method may be used to show how this relates to a formal written method.
<table><tr><th>x</th><th>300</th><th>20</th><th>7</th></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</td>
<td>327
x 4
28
80
1200
1308
This may lead to a compact method.

</td>
</tr>
</table>

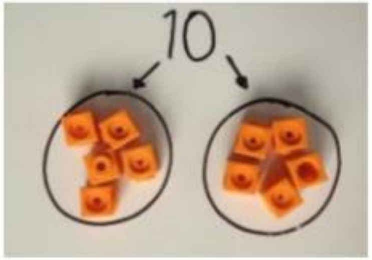
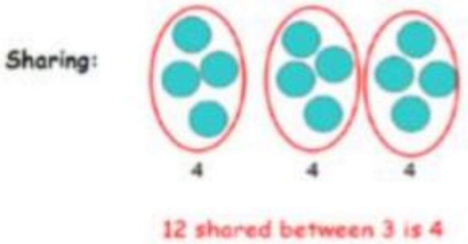
Year 5- 6 Multiplication

Objective and strategy	Concrete	Pictorial	Abstract
Column multiplication for 3 and 4-digits by 1 digit.	It is important at this stage that they always multiply the ones first.  Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	The grid method may be used to show how this relates to a formal written method. x | 300 | 20 | 7 4 | 1200 | 80 | 28	327 x 4 28 80 1200 1308 This may lead to a compact method. 3 2 7 × 4 1 3 0 8 1 2
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	Continue to use bar modelling to support problem solving.  	 1 2 3 4 × 1 6 7 4 0 4 (1234 × 6) 1 2 3 4 0 (1234 × 10) 1 9,7 4 4
Year 6 - Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and then answer. 3 · 1 9 × 8 2 5 · 5 2

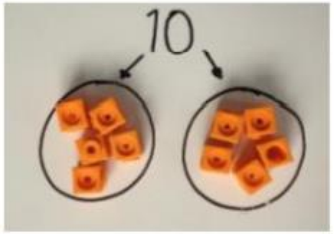
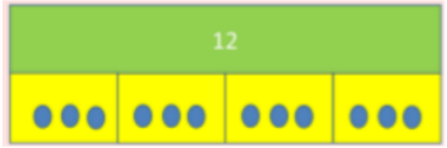
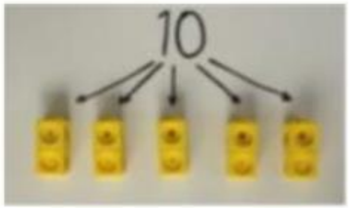
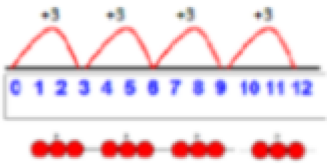
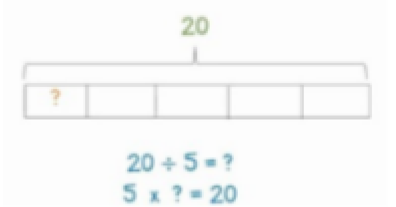
Nursery and EYFS Division

Objective and Strategy Models and Guidance	Objective and Strategy Models and Guidance
To solve problems, including halving and sharing.	**Nursery and Reception:** By the end of Reception, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities such as sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.         12 shared equally by 3 is 4 

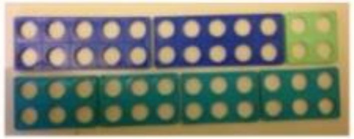
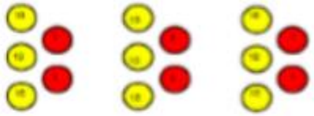
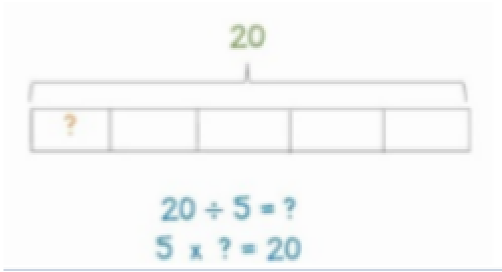
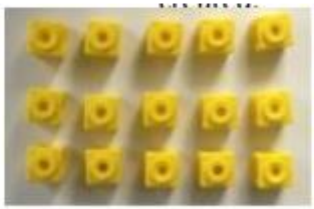
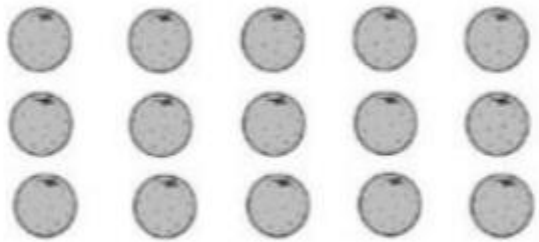
Year 1 Division

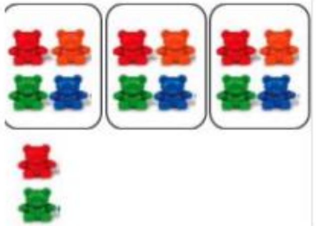
Objective and strategy	Concrete	Pictorial	Abstract
Division as sharing. *Use ITPs for modelling.*	Use cubes, counters and real-life objects to share.   I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures to share different quantities.  8 shared between 2 is 4  	12 shared between 3 is 4

Year 2 Division

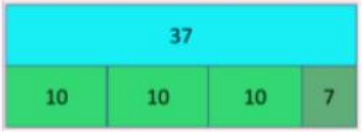
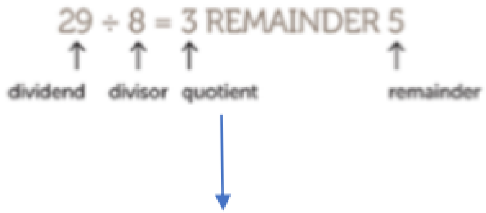
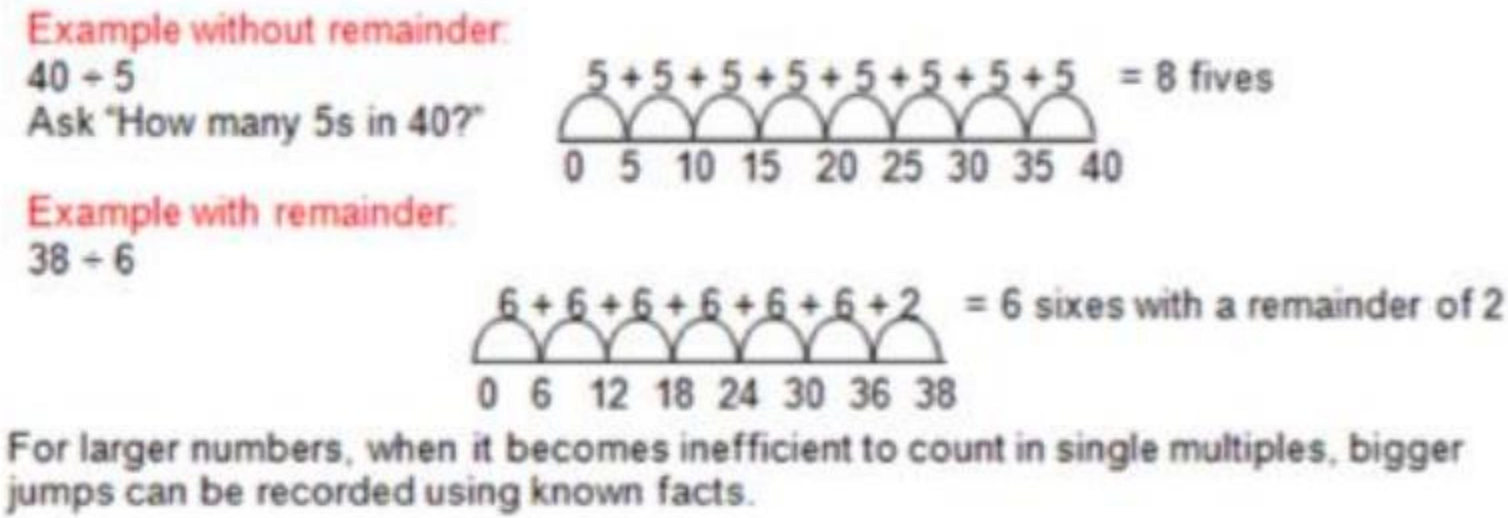
Objective and strategy	Concrete	Pictorial	Abstract
Division as sharing.	I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures to share different quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 4 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping. 12 ÷ 4 = 3  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Year 3 Division

Objective and strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. *24 divided into groups of 6 = 4*  96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7

<table>
<tr>
<td>Division with remainders</td>
<td>Divide objects between groups and see how much is left over.
$14 \div 3$
</td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.

Draw dots and group them to divide an amount and clearly show a remainder.

Use bar models to show division with remainders.
</td>
<td>Complete written divisions and show the remainder using the notation of ‘r’.

$29 \div 8 = 3$ r 5</td>
</tr>
<tr>
<td colspan="4"></td>
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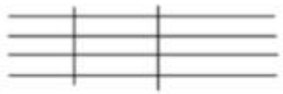
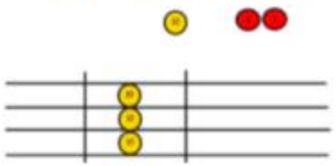
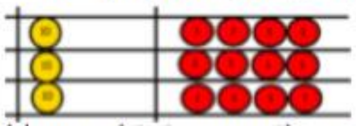
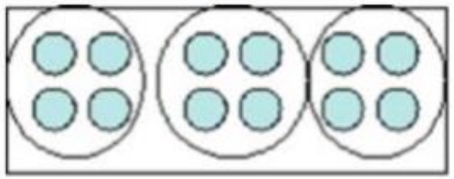
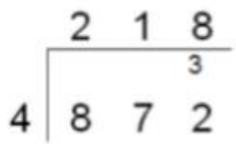
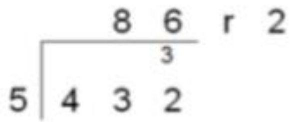
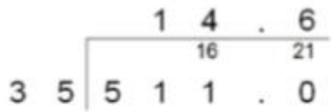
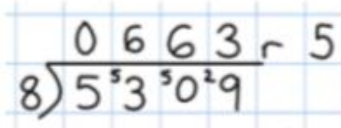
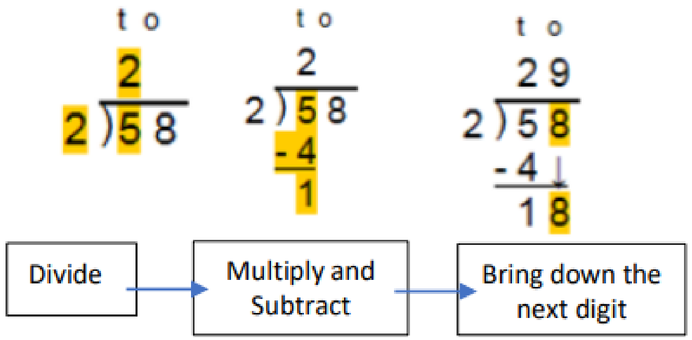
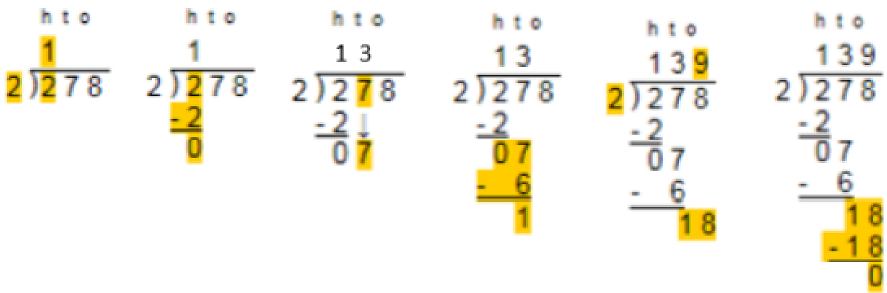
Year 4-6 Division

Objective and strategy	**Concrete**	**Pictorial**	**Abstract**
Divide at least 3-digit numbers by a 1-digit number. Short Division.	 Use place value counters to divide using the bus stop method alongside. *42 ÷ 3*   Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage children to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainders.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Long division with remainders in the ones.	h t o 1. 4 does not go into 1 (hundred) so we combine the 1 hundred with the 6 tens (160) 2. 4 goes into 16 (tens) 4 times (16 ÷ 4 = 4) 3. 4 goes into 5 once, leaving a reminder of 1.  1. 8 does not go into 3 (thousands) so we combine the 2 thousands with the 2 hundreds (3200) 2. 8 goes into 32 (hundreds) 4 times (32 ÷ 8 = 4) 3. 8 goes into 0 zero times (tens). 4. 8 goes into 7 zero times, and leaves a remainder of 7.
Long division with remainders in the tens.	 1. 2 goes in 5 twice (5 ÷ 2 = 2) but there is a remainder. 2. To find the remainder multiply 2 by 2= 4. Subtract the 2 from the 5 (tens) to find the remainder of 1 (ten). 3. Next, bring the 8 down next to the 1 (ten) left over. 4. 2 goes in 18, 9 times (18 ÷ 2 = 9).
Long division with a remainder in any place value column.	 1. 2 goes into 2 once (2 ÷ 2 = 1). 2. Multiply 1 x 2 = 2. Subtract the 2 to leave you with a remainder of 0. 3. Bring down the 7 from the tens and place next to the 0. 4. Divide 7 by 2. Place 3 in the quotient. 5. Multiply 3 x 2 = 6. Subtract the 6 to leave you with a remainder of 1. 6. Bring down the 8 from the ones and place next to the 1. 7. Divide 18 by 2. Place 9 in the quotient. 8. There are no more digits to drop down. The quotient is 139.